z-Transform Practice Problems Solutions

1. Consider the following LCCDE:

$$y[n] - 2y[n-2] = x[n] - 2x[n-1] + x[n-2]$$

   (a) What is the transfer function $H(z)$ for this system?

$$Y(z) - 2z^{-2}Y(z) = X(z) - 2z^{-1}X(z) + z^{-2}X(z)$$

$$Y(z)(1 - 2z^{-2}) = X(z)(1 - 2z^{-1} + z^{-2})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 2z^{-1} + z^{-2}}{1 - 2z^{-2}}$$

   (b) What are the poles and zeros?

$$\begin{aligned} H(z) &= \frac{z^2}{z^2}\frac{1 - 2z^{-1} + z^{-2}}{1 - 2z^{-2}} \\ &= \frac{z^2 - 2z + 1}{z^2 - 2} \\ &= \frac{(z-1)(z-1)}{(z-\sqrt{2})(z+\sqrt{2})} \end{aligned}$$

   (c) Is this system causal?

   Yes, it only uses values of x and y in the past.

   (d) Is it stable?

   No, the poles $\pm\sqrt{2}$ are outside the unit circle.

2. Consider an LTI system with transfer function:

$$H(z) = \frac{z+1}{z^2 - 0.25}.$$

   (a) What is an LCCDE for this system?

$$\begin{aligned} H(z) &= \frac{z^{-2}}{z^{-2}}\frac{z+1}{z^2 - 0.25} \\ &= \frac{z^{-1} + z^{-2}}{1 - 0.25z^{-2}} \end{aligned}$$

   Now, using the time shift rule that z^{-k} corresponds to a time shift of $n-k$, we can read off the x terms of the LCCDE from the numerator, and the y terms from the denominator to get:

$$y[n] - 0.25y[n-2] = x[n-1] + x[n-2]$$

(b) What are the poles and zeros?

Poles: $z = \pm\sqrt{0.25} = \pm 0.5$
Zeros: $z = -1$

(c) Is this system stable?

Yes, poles are inside the unit cirle (magnitude < 1).