ECE 4750/6750 + CS 4501/6501: Digital Signal Processing

Final Exam Practice Problems

1. For the following signals, write if they are "periodic" or "aperiodic". If they are periodic, also give their frequency and period.
   (a) $x[n] = \cos\left(\frac{4n}{5}\right)$
   (b) $x[n] = \sin\left(\frac{12\pi n}{7}\right)$
   (c) $x[n] = e^{i3\pi n}$
   (d) $x[n] = 3^n e^{\frac{2\pi n}{3}}$

2. Consider the following systems:

$$T_1\{x[n]\} = x[2n+1] \qquad T_2\{x[n]\} = \sin(x[n]) \qquad T_3\{x[n]\} = 3$$

$$T_4\{x[n]\} = \sum_{k=-\infty}^{n} x[k] \qquad T_5\{x[n]\} = \sum_{k=1}^{\infty} \frac{1}{2^k} x[n-k] \qquad T_6\{x[n]\} = x[n] * x[n]$$

   (a) List which of these systems are linear.
   (b) List which of these systems are time-invariant.
   (c) List which of these systems are LTI and BIBO stable.
   (d) List which of these systems are causal.
   (e) For each system, sketch the impulse response function for each of these systems in the range $-4 \leq n \leq 4$.

3. Consider two periodic signals, $x_1[n]$ and $x_2[n]$, both with period $L = 10$, given by:

$$x_1[n] = \cos(\pi n), \qquad x_2[n] = \sin(\pi n).$$

   (a) What is ω_0 for these two signals?
   (b) What are their DFTs, $X_1[k]$ and $X_2[k]$?
   (c) What is $\mathcal{DFT}\{x_1[n] * x_2[n]\}$? This should be a function of k only. Write out all known constants (for example, don't use the symbol L, but rather plug in its value $L = 10$, etc.).
   (d) What is $x_1[n] * x_2[n]$? Again,this should be a function of k, and write out all known constants rather than leaving the corresponding symbols.
   (e) If these signals came from sampling a continuous signal with sampling period $T = 0.04$ seconds, what is the frequency in Hertz represented by the bin $k = 3$ in the DFT?

4. Consider the following LCCDE:

$$y[n] = x[n] + 2.0x[n-1] + y[n-1] - \frac{1}{2}y[n-2]$$

   (a) Write down the transfer function, $H(z)$, for this system.
   (b) Sketch a pole-zero plot for this system.
   (c) Is this a causal system?
   (d) What is the region of convergence for this system?
   (e) Is this a BIBO stable system?

(f) Which best describes the frequency response: low-pass, high-pass, band-pass, or all-pass? Explain why.

(g) What is transfer function for the inverse system?

(h) Sketch a pole-zero plot for the inverse transfer function.

(i) What region of convergence makes the inverse BIBO stable? (say "none" if it isn't possible)

(j) Is this BIBO stable system also causal? (say "not applicable" if previous answer was "none")

(k) Write an LCCDE corresponding to the inverse transfer function. (There are multiple correct answers depending on the ROC you choose.)

5. Consider two continuous signals, $x_1(t), x_2(t)$, with corresponding Fourier transforms $X_1(\Omega), X_2(\Omega)$, that satisfy:

$$|X_1(\Omega)| = 0, \text{for } |\Omega| > 12\pi, \qquad |X_2(\Omega)| = 0, \text{for } |\Omega| > 5\pi.$$

(a) Assume both signals were sampled with a time period of $T = 0.1$ seconds. Is $x_1(t)$ being sampled above the Nyquist rate? Is $x_2(t)$?

(b) Consider the signal $x_1(t) * x_2(t)$. What is the Nyquist sampling rate (in Hertz) needed to faithfully represent it?

(c) Consider the signal $x_1(t)x_2(t)$. What is the Nyquist sampling rate (in Hertz) needed to faithfully represent it?

6. Consider a stable LTI system with transfer function

$$H(z) = \frac{1 + 4z^{-2}}{1 - \frac{1}{4}z^{-1} - \frac{3}{8}z^{-2}}.$$

(a) Sketch a pole-zero diagram for this system.

(b) What is the ROC for this system? Sketch it on the pole-zero diagram.

(c) Factorize this system into a product of an all-pass system, $H_{\text{AP}}(z)$, and a minimum-phase system, $H_{\text{min}}(z)$, i.e., $H(z) = H_{\text{AP}}(z)H_{\text{min}}(z)$. Write equations for the system functions $H_{\text{AP}}(z)$ and $H_{\text{min}}(z)$.

(d) Draw pole-zero diagrams for $H_{\text{AP}}(z)$ and $H_{\text{min}}(z)$. Sketch the ROC for both systems in the diagrams.

7. Determine if the following statements are true. If a statement is true, give a concise arguments for why. If it is false, give a system that provides a counterexample.

(a) The transfer function for a causul, finite impulse response filter must have a pole at zero with multiplicity k, where k is the largest integer for which there is an $x[n-k]$ term in the corresponding LCCDE.

(b) The transfer function for an infinite impulse response filter must have a pole somewhere in the z-plane away from the origin.

(c) A finite impulse reponse filter will always have linear phase.

(d) The transfer function for a zero-phase filter has poles only at the origin (or no poles).