Perceptron

Foundations of Data Analysis

April 12, 2022

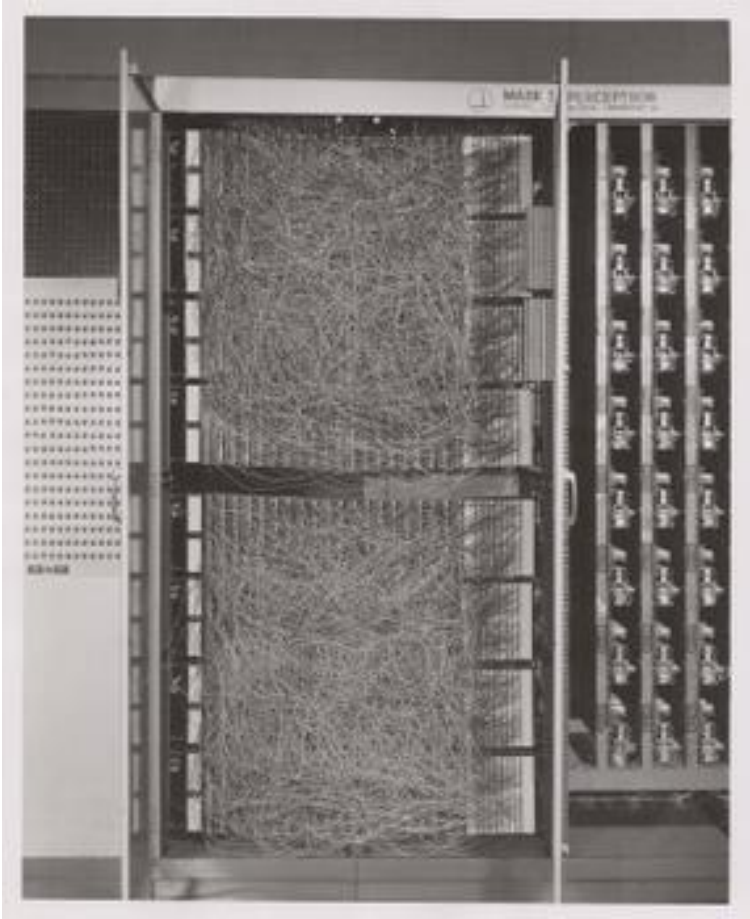
History of Perceptron

- Frank Rosenblatt
- 1928-1969

invented perceptron algorithm

History of Perceptron

- Mark 1 Perceptron (1958)
- 20 x 20 pixel camera
- Hardware, not software!



"an electronic computer that [the Navy] expects will be able to walk, talk, see, write, reproduce itself and be conscious of its existence"

- NY Times, 1958

Perceptron Learning Algorithm

- First neural network learning model in the 1960's

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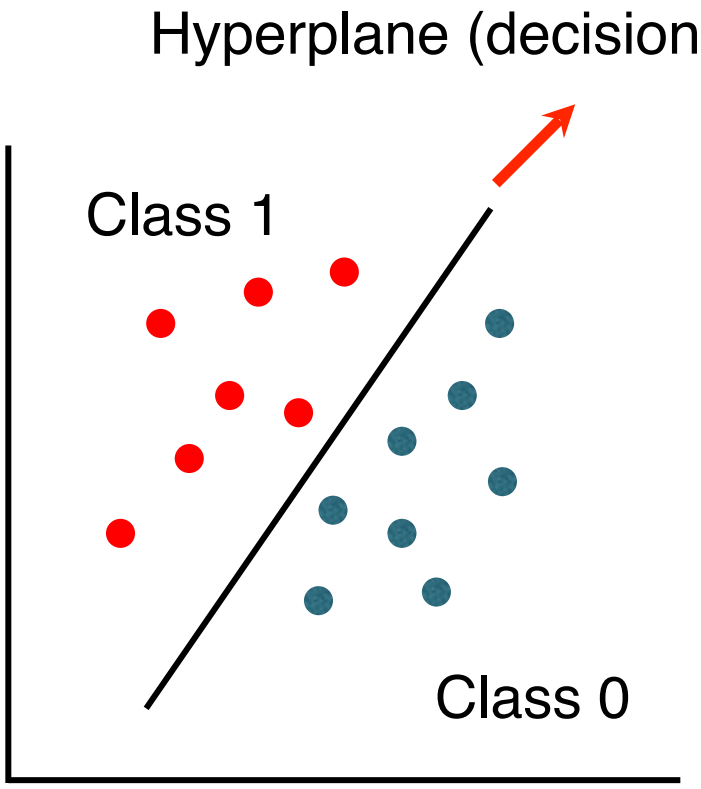
- Simple and limited (single layer model)

Perceptron Learning Algorithm

- First neural network learning model in the 1960's
- Simple and limited (single layer model)
- Basic concepts are similar to multi-layer models

What is Perceptron?

The goal of perceptron algorithm is to find a hyperplane that separates a set of data into two classes.



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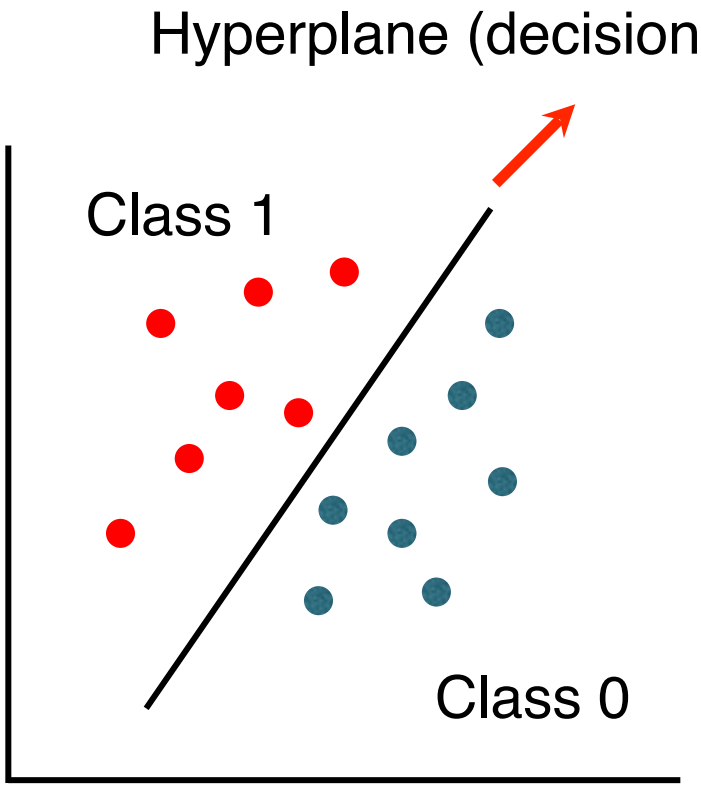
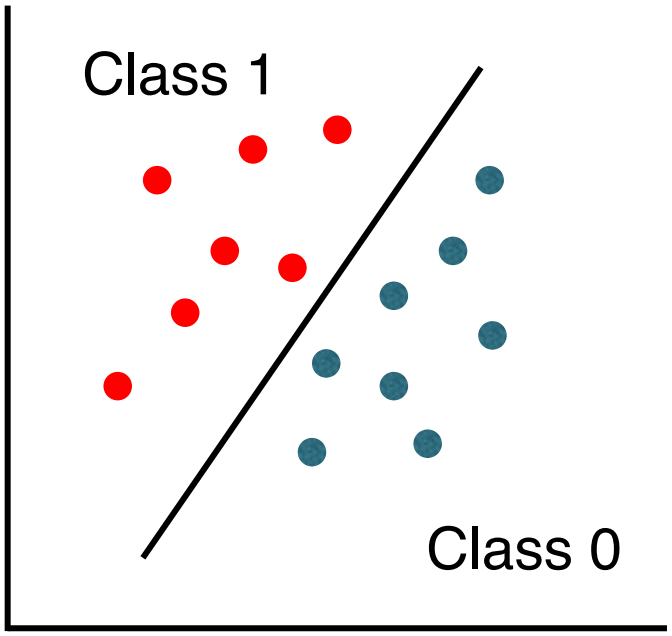


- Binary classifier
- Supervised learning

Perceptron

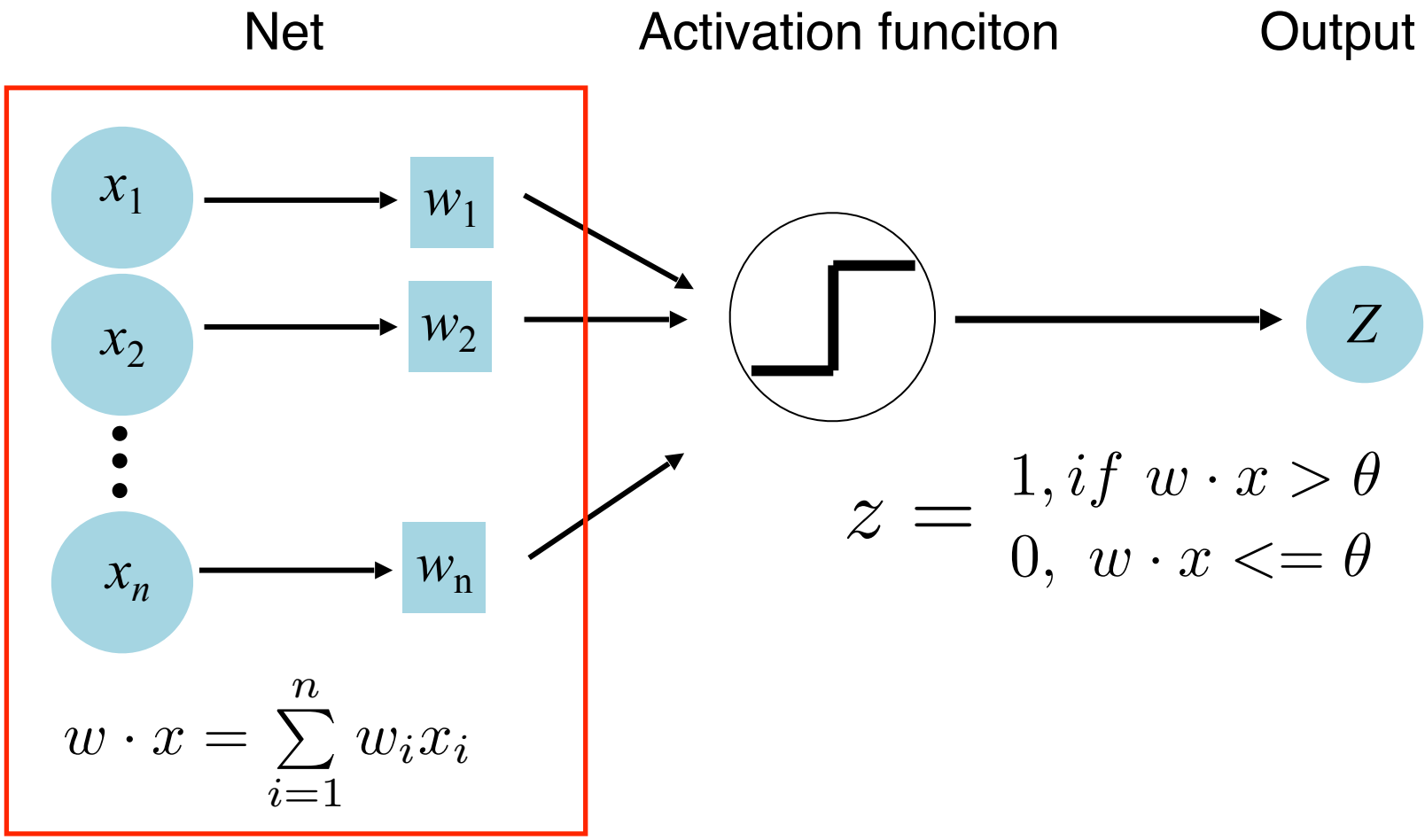


bias term

$$f(x) = \begin{matrix} 1, if\ w \cdot x + \theta > 0 \\ 0, otherwise \end{matrix}$$

$$w \cdot x = \sum_{i=1}^{n} w_i x_i \quad \text{(dot product)}$$

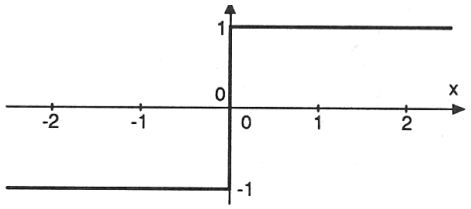
Perceptron



- Learning weights such that an objective function is minimized

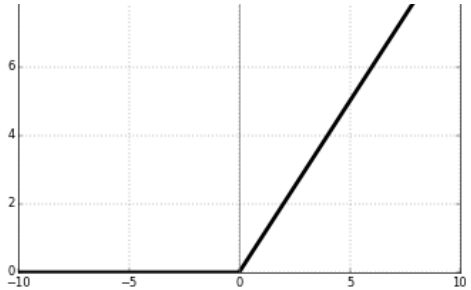
Activation Function

Outputs the label given an input or a set of inputs.



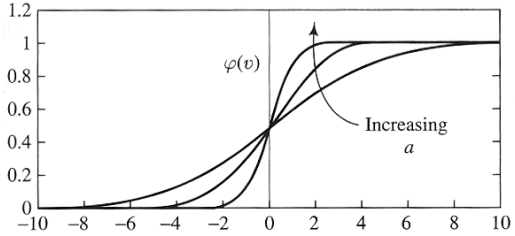
Step function

$$f(x) := \mathrm{sgn}(x) = \begin{cases} 1 & if \quad x \geq 0 \\ -1 & if \quad x < 0 \end{cases}$$



ReLU (rectified linear unit)

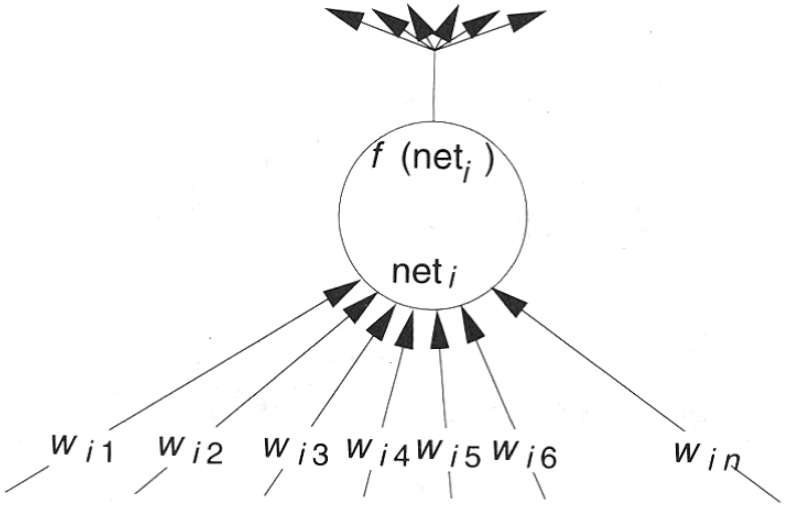
$$f(x) = max(0, x)$$

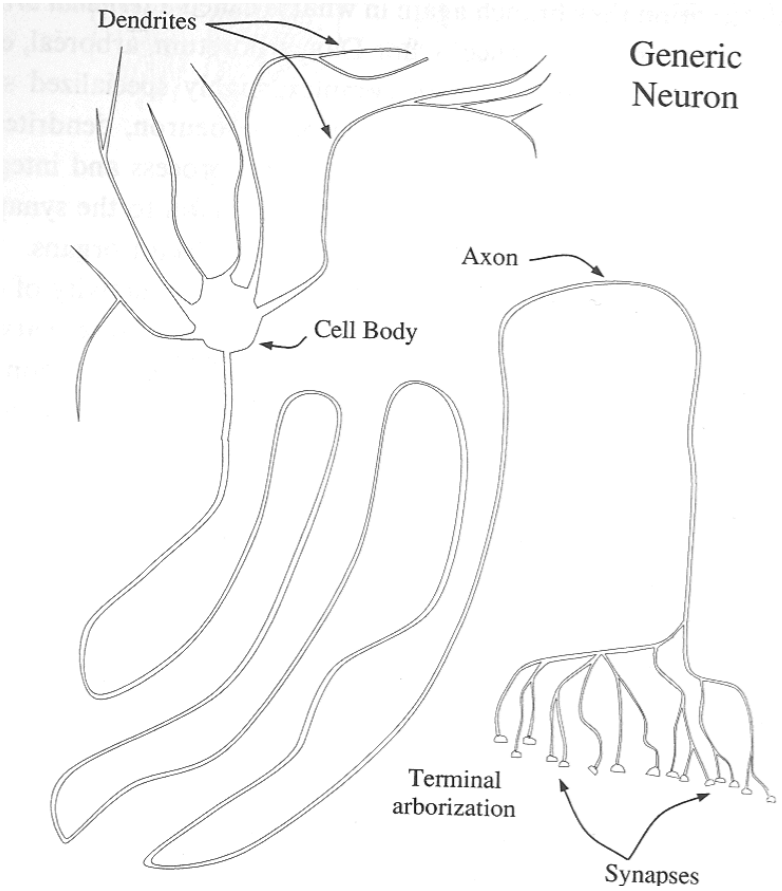


Sigmoid function

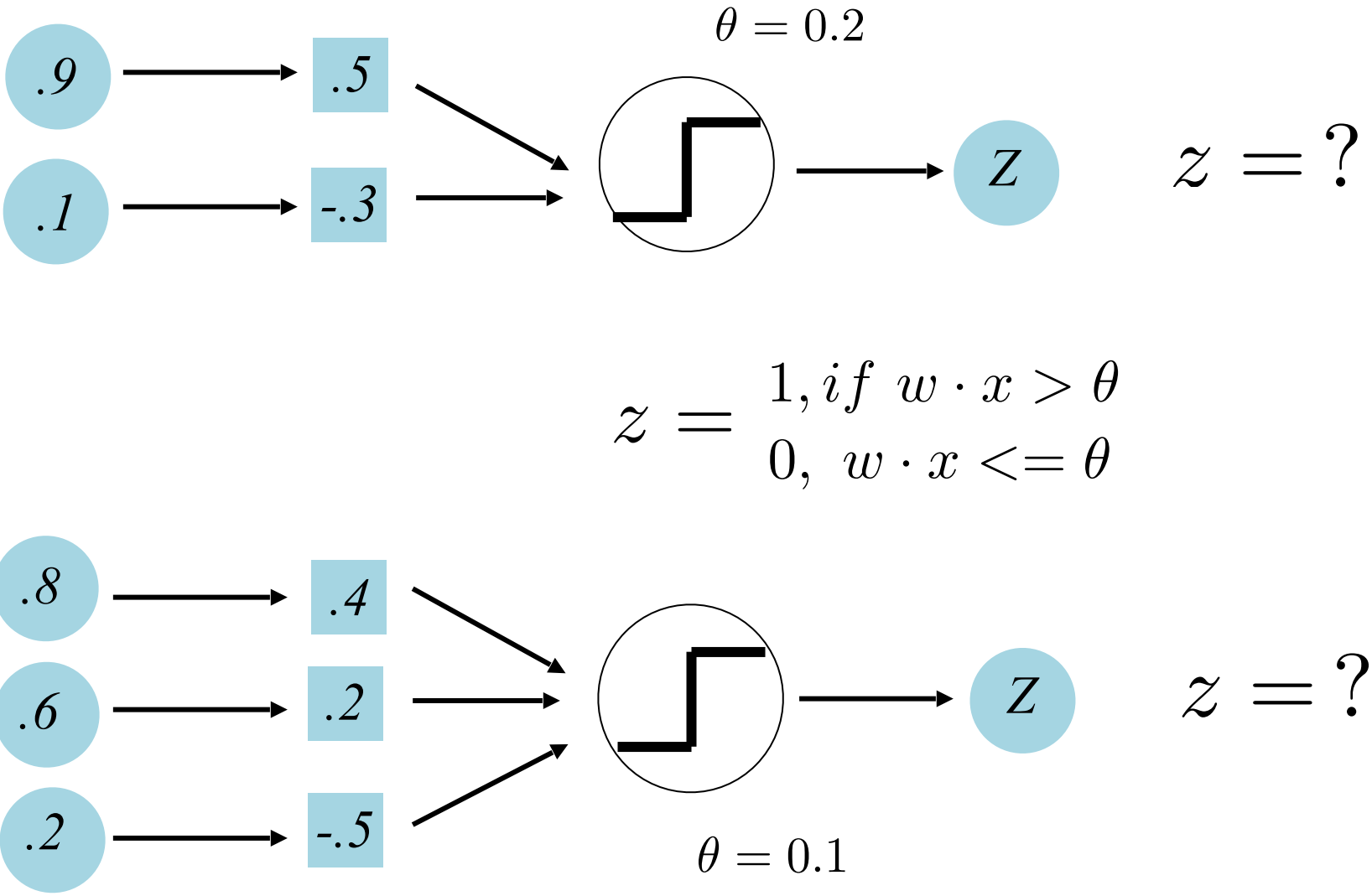
$$f(x) := \sigma(x) = \frac{1}{1 + e^{(-ax)}}$$

Perceptron as a Single Layer Neuron

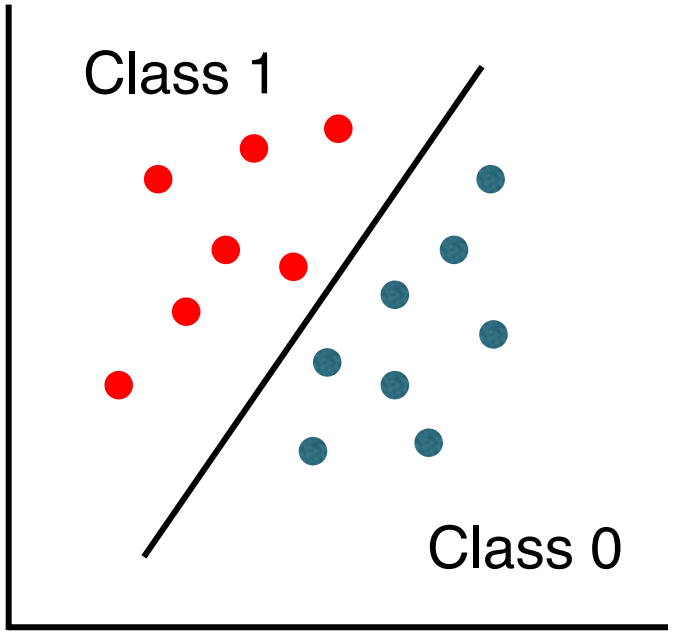




Examples


.9
.1
.5
-.3
$\theta = 0.2$
z
$z = ?$
$z = \begin{cases} 1, if\ w \cdot x > \theta \\ 0,\ w \cdot x <= \theta \end{cases}$
.8
.6
.2
.4
.2
-.5
z
$z = ?$
$\theta = 0.1$

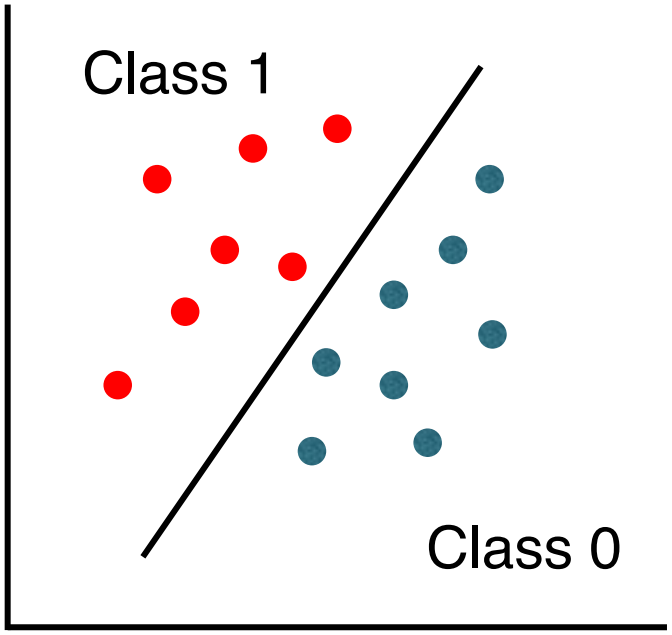
How to Learn Perceptron?



$$f(x) = \begin{matrix} 1, if\ w \cdot x + \theta > 0 \\ 0, otherwise \end{matrix}$$

w, θ are unknown parameters

How to Learn Perceptron?

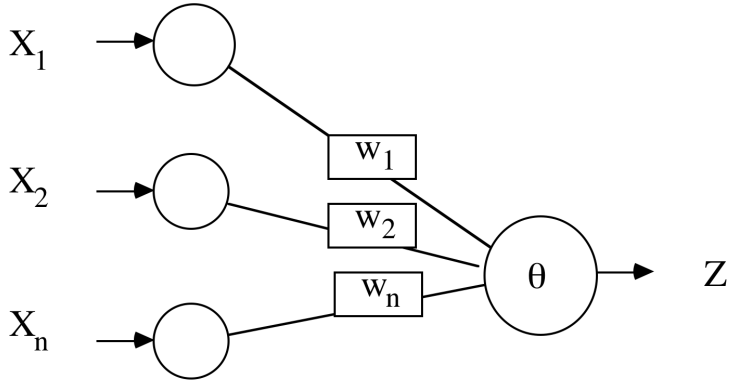


$$f(x) = \begin{matrix} 1, if\ w \cdot x + \theta > 0 \\ 0, otherwise \end{matrix}$$

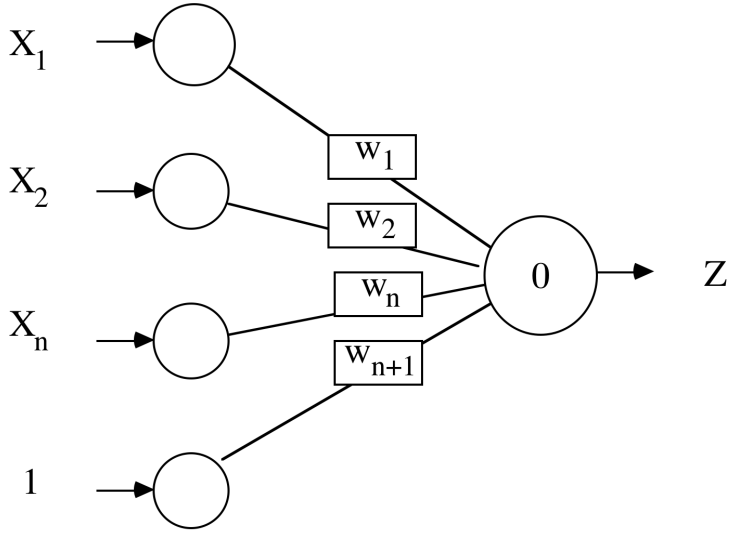
w, θ are unknown parameters

- In supervised learning the network has its output compared with known correct answers
  - Supervised learning
  - Learning with a teacher

Weight Versus Threshold



Do you need to adjust Theta? Yes, in most cases



where $w_{n+1} = -\theta$

Perceptron Learning Rules

- Consider linearly separable problems
- How to find appropriate weights
- Look if the output result *o* belongs to the desired class has the desired value *d* (give labels)

$$w^{new} = w^{old} + \triangle w \quad \triangle w = \eta \sum_i (d - o) x_i$$

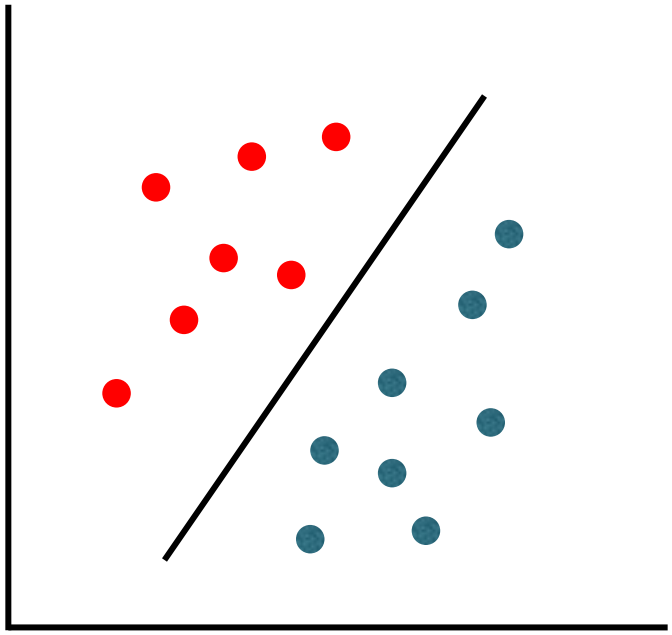
η is called the learning rate, with $0 < \eta \leq 1$

Perceptron Convergence Theorem: Guaranteed to find a solution in finite time if a solution exists

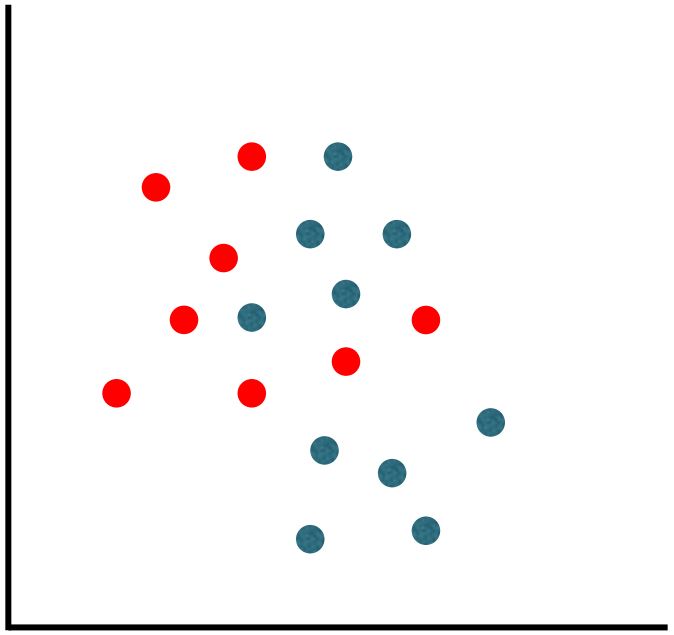
Perceptron Learning Rules

- The algorithm converges to the correct classification *if and only if* the training data is linearly separable
- When assigning a value to η we must keep in mind two conflicting requirements
  - Averaging of past inputs to provide stable weights estimates, which **requires small η**
  - Fast adaptation with respect to real changes in the underlying distribution, which **requires large η**

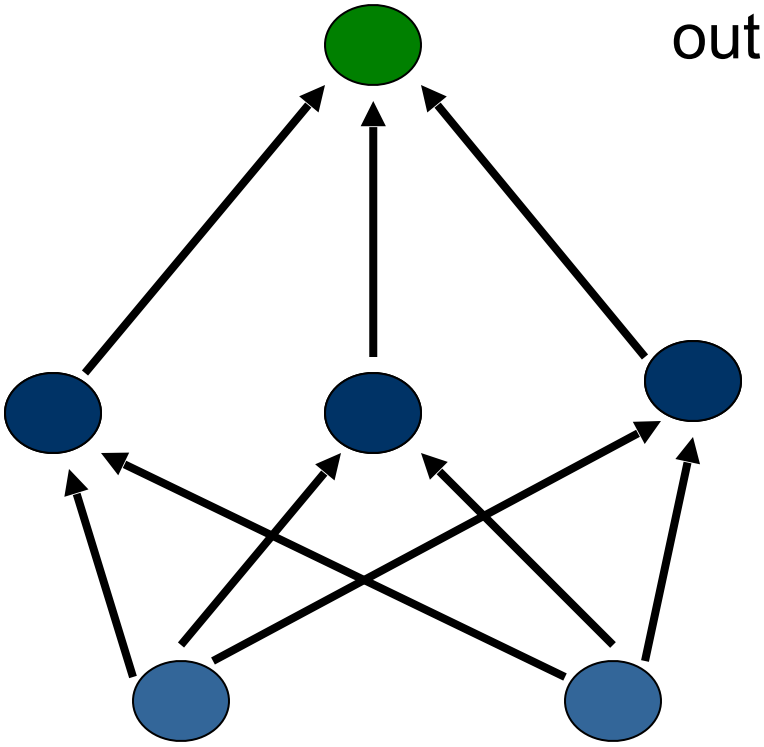
Linear Separability



Limited Functionality of Hyperplane



Multilayer Network



output layer

$$o_1 = \mathrm{sgn}(\sum_{i=0}^{n} w_{1i} x_i)$$

hidden layer

$$o_2 = \mathrm{sgn}(\sum_{i=0}^{n} w_{2i} x_i)$$

input layer