# Probability Cheat Sheet

Foundations of Data Analysis

**Review of "English translation" for events:**

- $A \cap B =$ "both events $A$ and $B$ happen"
- $A \cup B =$ "either event $A$ or $B$ (or both) happens"
- $A^c =$ "event $A$ does not happen"
- $A - B = A \cap B^c =$ "event $A$ happens, but event $B$ does not happen"

**Set Theory Rules**: (try drawing Venn diagrams of these)

- Associative Law:
$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$
- Commutative Law:
$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$
- Distributive Law:
$$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$$
$$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$$
- DeMorgan's Law:
$$(A \cup B)^c = A^c \cap B^c$$
$$(A \cap B)^c = A^c \cup B^c$$

**Counting:**

- **Number of permutations of $n$ items:** $n! = n \times (n-1) \times (n-2) \times \cdots \times 2$
**(number of unique orderings)**
- **Number of ways to select $k$ items out of $n$ choices:** $\binom{n}{k} = \frac{n!}{k!(n-k)!}$
**(here order does not matter, just which $k$ items you select)**

**Probability Rules:**

- **Equally likely outcomes:** $P(A) = \frac{|A|}{|\Omega|}$
- **Inclusion-Exclusion Rule:** $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- **Complement Rule:** $P(A^c) = 1 - P(A)$
- **Difference Rule:** $P(A - B) = P(A) - P(A \cap B)$

Exercise: Try deriving these rules from the definition of a probability function. Draw a Venn diagram to convince yourself they work.

**Conditional Probability:**

$$P(A \mid B) = \text{“the probability of event } A \text{ given that we know } B \text{ happened”}$$

Formula:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

**Multiplication Rule:**

$$P(A \cap B) = P(A \mid B)P(B)$$

**Total Probability:**

$$P(A) = P(A \mid B)P(B) + P(A \mid B^c)P(B^c)$$

**Bayes' Rule:**

$$P(B \mid A) = \frac{P(A \mid B)P(B)}{P(A)}$$

Note: The denominator, $P(A)$, in Bayes' Rule is often computed using the Total Probability formula above!

**Steps to Solve a Word Problem:**

1. Identify the events in the problem.
2. In terms of these events, write down the probability that is being asked for in the problem.
3. For each probability (number) in the problem, write down what its mathematical expression is, e.g., $P(A), P(B^c), P(A \mid B)$, etc.
4. Use the formula for the probability being asked. The right hand side should all be known, or computable, from what is given in the problem.